Spring 2006 Special Focus–Discrete Applied Mathematics II

MATH 554: Probabilistic Methods in Discrete Applied Mathematics

Spring Semester, MW 12:25-1:40pm

Instructor: Robert Ellis, rellis@math.iit.edu

Paul Erdős, the most prolific discrete mathematician of the 20th century, pioneered the probabilistic method in the 1950's. Rather than directly proving the existence of a combinatorial object with certain desired properties, we can define a probability space on a set of combinatorial objects and prove that one with the properties we want exists with positive probability. The probabilistic method has become a crucial tool for mathematicians and computer scientists in many areas: graph coloring and Ramsey Theory, packing and coverings, coding theory, combinatorial number theory, random graphs and internet modelling, and randomized algorithms. The powerful joining of probability theory and combinatorial perspective has led to an explosion of research employing the probabilistic method in the last two decades.

This course is a graduate-level introduction to the basic probabilistic methods: linearity of expectation, the deletion method, the second moment method, and the Lovász Local Lemma. Typically these methods define a random variable on a combinatorial probability space and deduce properties of the space from its expected value, variance or other characteristics. We will illustrate these methods with recent active research in internet source detection, coding theory, and modelling ad hoc wireless networks.

Course objectives. To confer a firm basis in the probabilistic method in order to enable the student to better understand or participate in probabilistic methods research and applications. To prepare the student for future study of advanced probabilistic methods.

Text. The primary text will be *The Probabilistic Method*, by N. Alon and J. Spencer (2nd ed. 2000), supplemented by other texts and research articles.

Prerequisites. Graduate status or consent of the instructor. Graduate students in applied mathematics, computer science or engineering are especially encouraged to enroll. Experience with proofs or "mathematical maturity" is advisable. Undergraduates with evidence of high performance (e.g., A's in upper division courses) will be admitted by consent of the instructor.

Course Title: Probabilistic Methods in Discrete Applied Mathematics – Special Focus Semester of Math 554

Instructor: Robert Ellis, Assistant Professor of Applied Mathematics

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Course Description: This course is a graduate-level introduction to the basic probabilistic methods: linearity of expectation, the deletion method, the second moment method, and the Lovász Local Lemma. Typically these methods define a random variable on a combinatorial probability space and deduce properties of the space from its expected value, variance or other characteristics. Often an object or structure with desired properties is shown to exist with positive probability. The basic probabilistic methods will be illustrated with recent active research in internet source detection, coding theory, and modelling ad hoc wireless networks. More advanced techniques such as martingales, and FKG, Talagrand and Janson inequalities will be treated as time permits.

Topics:	**Methods**	**Applications**
	Linearity of expectation/First moment method	Ramsey Theory
	Deletion Method	Coding theory
	Second moment method	Random geometric graphs
	Lovász Local Lemma	Internet source detection
	Derandomization	Randomized algorithms

Primary Text: *The Probabilistic Method*, N. Alon and J. Spencer, 2nd ed., 2000.

Supplementary Material: At the discretion of the instructor, possibly including:

M. Molloy and B. Reed, *Graph colouring and the probabilistic method*, 2002.

R. L. Graham, B. L. Rothschild and J. H. Spencer, *Ramsey theory*, 2nd ed., 1990.

F. Chung and R. Graham, *Erdős on graphs*, A K Peters, 1998.

J. Spencer, *The strange logic of random graphs*, Springer, Berlin, 2001.

F. Chung, R. Graham and T. Leighton, "Guessing Secrets," Electron. J. Combin. **8** (2001), no. 1.

M. Krivelevich, B. Sudakov and V. H. Vu, "Covering codes with improved density," IEEE Trans. Inform. Theory **49** (2003), no. 7.

R. B. Ellis, J. L. Martin and C. H. Yan, "Random geometric graph diameter in the unit ball."

J. N. Cooper, R. B. Ellis and A. B. Kahng, "Asymmetric binary covering codes," J. Combin. Theory Ser. A **100** (2002), no. 2.

Course Objectives: The successful student in this course will do the following.

- Achieve a firm command of the basic probabilistic methods
- Apply these methods to Ramsey Theory and other combinatorial examples
- Explore applications of these methods to recent research
- Achieve a firm basis for understanding probabilistic methods publications
- Be able to apply probabilistic methods to personal research in mathematics, the sciences, or engineering