Homework 1

Recitation problems for Monday, 1/30/06

1. Prove the linearity of expectation formula for random variables $X, X_1, \ldots, X_n$ for finite probability spaces: if $X = \sum_{i=1}^n X_i$, then $E(X) = \sum_{i=1}^n E(X_i)$.

2. Prove the conditional version of linearity of expectation: if $X = \sum_{i=1}^n X_i$ and B is an event, then $E(X|B) = \sum_{i=1}^n E(X_i|B)$.

3. Prove the following conditional reduction formula: If Ω is partitioned by events $B_1, \ldots, B_l$, and X is a random variable on Ω, then $E(X) = \sum_{i=1}^l E(X|B_i)\Pr(B_i)$.

4. Let $G_{n,p}$ be the Erdős-Rényi random graph model, which can be viewed as a product space over all possible $\binom{n}{2}$ edges of the graph. Assume n is even, and label the vertices $v_1, \ldots, v_n$. Let B be the event that edges $v_1v_2, v_3v_4, \ldots, v_{n-1}v_n$ are present. Compute the expected value of the number of 4-cliques, or complete subgraphs on 4 vertices.

5. Let n and k be arbitrary positive integers, and $c \in [0, n]$. Let $X = \sum_{i=1}^n$ where the X_i's are independent random indicator variables with $X_i = 1$ with probability c/n and 0 otherwise. Prove $\Pr(X \geq k) \leq c^k/k!$.

Written problems for Wednesday, 2/1/06

1. (Not to turn in). Review "Big Oh" notation, from a source such as `http://en.wikipedia.org/wiki/Big_O_notation`. We will frequently use the notations $O(\cdot)$, $o(\cdot)$, $\Omega(\cdot)$, $\omega(\cdot)$ and $\Theta(\cdot)$.

2. Exercise 1, p. 10, Alon and Spencer.

3. Exercise 2, p. 10, Alon and Spencer.