Homework 2

Recitation problems for Monday, 2/6/06

1. For this problem compute an upper bound on the size of a covering code $\mathcal{C}$ of length n and radius R. $\mathcal{C}$ is such a code if
   - $\mathcal{C} \subseteq Q_n := \{0,1\}^n$
   - For every $v \in Q_n$, there exists a $c \in \mathcal{C}$ such that v can be obtained from c by flipping at most R bits.

   Instead of selecting each vertex to be in $\mathcal{C}$ individually, fix a k and select a random subset of Q_n of size k. Can you find a covering code of size k this way?

2. The definitions are just as in #1. This time, however, use the deletion method. In other words, select a subset of size k, and include that subset and all uncovered vertices of Q_n to be in $\mathcal{C}$. How small of a covering code can you get in this way? How does the size compare to the method in class in which every vertex is chosen with fixed probability?

3. Use the Stirling approximation
$$n! = \sqrt{2\pi n}\left(\frac{n}{e}\right)^n \left[1 + \frac{1}{12n} + O\left(\frac{1}{n^2}\right)\right]$$
   to improve the lower bound on the diagonal Ramsey number $R(k,k)$.

4. Prove the diagonal Ramsey number bounds
$$R(k,t) \leq \binom{k+t-2}{k-1}$$
   and
$$R(k,k) \leq \binom{2k-2}{k-1} \approx \frac{c2^{2k-2}}{\sqrt{k-1}}.$$
   From this and the lecture notes deduce lower and upper bounds on $(R(k,k))^{1/k}$.

Written problems for Wednesday, 2/8/06

1. The Erdős-Rényi random graph $G(n,p)$, find the expected number of neighbors of a fixed vertex. Find p so that the expected number of neighbors is $\ln n$.

2. The random geometric graph $G(n,\lambda)$ in the unit disk is defined as follows. Randomly select n vertices in the disk $\mathcal{D} := \{x : \|x\|_2 \leq 1\}$, each independently and from the uniform distribution over $\mathcal{D}$ (technically, under Lesbesgue measure). Find an approximation for the expected number of neighbors of a fixed vertex, being careful of what happens near the boundary. Find λ so that the expected number of neighbors is approximately $\ln n$.

3. Interpret the two results in #1 and #2: find a way of interpreting the λ in $G(n,\lambda)$ so that the value of λ is analogous to the value of p.