Homework 3

Recitation problems for Monday, 2/13/06

1. For this problem compute an upper bound on the size of a covering code $\mathcal{C}$ of length n and radius R. $\mathcal{C}$ is such a code if
   - $\mathcal{C} \subseteq Q_n := \{0,1\}^n$
   - For every $v \in Q_n$, there exists a $c \in \mathcal{C}$ such that v can be obtained from c by flipping at most R bits.

   Instead of selecting each vertex to be in $\mathcal{C}$ individually, fix a k and select a random subset of Q_n of size k. Can you find a covering code of size k this way?
2. The definitions are just as in #1. This time, however, use the deletion method. In other words, select a subset of size k, and include that subset and all uncovered vertices of Q_n to be in $\mathcal{C}$. How small of a covering code can you get in this way? How does the size compare to the method in class in which every vertex is chosen with fixed probability?
3. Problem 4, p. 11 of Alon and Spencer.
4. Problem 7, p. 11 of Alon and Spencer.
5. Problem 10, p. 11 of Alon and Spencer
6. Show $R(3,t) > t^{3/2+o(1)}$ using the deletion method. What happens using the first moment method?

Written problems for Wednesday, 2/8/06

1. Problem 8, p. 11 of Alon and Spencer.
2. This problem is similar to problem 1, p. 10 of Alon and Spencer. Give a lower bound for the Ramsey number $R(4,t)$ using the deletion method. In particular, show that for all p, $0 \le p \le 1$,

   $$R(4,t) \;>\; n - \binom{n}{4}p^6 - \binom{n}{t}(1-p)^{\binom{t}{2}}.$$

   Then optimize p to show that $R(4,t) = \Omega\left(\left(\frac{t}{\ln t}\right)^2\right)$.