Homework 5

Recitation problems for Monday, 2/27/06

1. Problems 4, 7, p. 11 of Alon and Spencer.
2. Problems 2, 3, 8, 9, p. 21 of Alon and Spencer.
3. Find $m = m(n)$ as large as you can so that the following holds: Let $A_1, \dots, A_m \subseteq \{1, \dots, 4n\}$ with all $|A_i| = n$. Then there exists a two coloring of $\{1, \dots, 4n\}$ such that none of the A_i are monochromatic. Use a random equicoloring of $\{1, \dots, 4n\}$ (i.e., $2n$ of each color). Express your answer as an asymptotic function of n.

Written problems for Wednesday, 3/1/06

1. Let G be a graph randomly chosen from the $G_{n,p}$ model. Let $k \geq 1$. Let A_{2k} be the event that G has an independent set of size $\lceil \frac{n}{2k} \rceil$ and let B be the event that G has at least $n/2$ triangles (distinct K_3 subgraphs). Prove that there exists an n such that $\Pr(A_{2k} \cup B) < 1$. Use Markov's inequality: for any nonnegative random variable X, $\Pr(X \geq t) \leq E(X)/t$. (An independent set is a subset of vertices with no edges between any pair of the subset.)
2. Choose one of these: #5 or #7 on p. 21 of Alon and Spencer.