Homework 7

Recitation problems for Monday, 3/20/06

1. Problems 3, 4, p. 36 of Alon and Spencer.
2. Problems 2, 3, 8, p. 21 of Alon and Spencer.
3. Problems 4, 7, p. 11 of Alon and Spencer.
4. Show that for any n sufficiently large, there exists a graph G on n vertices with chromatic number at least $n/2$ with clique number at most $n^{3/4}$. Chromatic number is the smallest number k such that the vertices of the graph can be partitioned into k parts with no edges inside any part. Clique number is the size of the largest clique (complete subgraph) in the graph. (Hint: What can you say about the chromatic number of the complement of a triangle-free graph?)

Written problems for Wednesday, 3/22/06

These problems will comprise the take-home midterm.

1. Find a lower bound for $m = m(n)$, as large as possible, such that there exists an $n \times n$ matrix with m 1's and $(n^2 - m)$ 0's containing no 3×3 submatrix with all 1's. Use the alteration method to get rid of bad submatrices. Optimize using calculus to get an answer of the form $m \sim an^b$.
2. Refer to p. 30 of Alon and Spencer (end of Section 3.4) for background. Let $Q_n = \{0,1\}^n$ be the binary discrete hypercube of dimension n. Let $B(x, R)$ be the hamming ball of radius R centered about $x \in Q_n$, and let $b_n(R) = \sum_{i=0}^{R} \binom{n}{i}$ be the size of $B(x, R)$. Now let $n \geq R_1 \geq R_2 \geq \cdots \geq R_m \geq 0$ be integers, and prove that there exists a packing in Q_n with m hamming balls of these respective radii whenever
$$\sum_{j=1}^{m} b_n(2 \cdot R_j) \;\leq\; 2^q \,.$$
(This is not an if and only if statement.) We discussed this in class, but I am looking for a formal proof.
3. Out of n couples ($2n$ total people) who purchased tickets for a flight, m people, picked uniformly at random, were denied boarding due to overbooking (of a couple, either person was equally likely to be denied or not denied). Compute the expected number of couples who successfully boarded the flight.