Homework 8

Recitation problems for Monday, 4/3/06

1. An *isolated triangle* in G is a triangle with no other edge connecting one of its three vertices to a vertex not in the triangle. Let X be the number of triangles in $G_{n,p}$ and find a prove a threshold function for G selected from $G_{n,p}$ having an isolated triangle. Use Chebyschev's Inequality and its corollaries.
2. Problems 2, 3, 5, 6 of Alon and Spencer.
3. Problem 4, p. 36 of Alon and Spencer.
4. Problems 3, 8, p. 21 of Alon and Spencer.
5. Problems 4, 7, p. 11 of Alon and Spencer.
6. Show that for any n sufficiently large, there exists a graph G on n vertices with chromatic number at least $n/2$ with clique number at most $n^{3/4}$. Chromatic number is the smallest number k such that the vertices of the graph can be partitioned into k parts with no edges inside any part. Clique number is the size of the largest clique (complete subgraph) in the graph. (Hint: What can you say about the chromatic number of the complement of a triangle-free graph?)

Written problems for Wednesday, 4/5/06

Choose 2 out of 3 problems.

1. Alon and Spencer, p. 58 #1.
2. Alon and Spencer, p. 59 #4.
3. The unit torus can be thought of as the unit square with opposite sides identified. A topological ball of radius r around a point close to the "boundary" in the unit torus will contain points on the "other side." The random geometric graph $G_\infty(n, \lambda)$ on the unit torus is defined as follows. Place n points uniformly and independently at random in the unit torus. Now connect any pair of points x, y with an edge provided their ℓ_∞ distance $\|x - y\|_\infty$ is at most λ. Find a threshold function for the property of having at least one isolated vertex. (Note: the ℓ_∞-ball about a point is a square.)