Homework 9

Recitation problems for Monday, 4/10/06

1. (Note: Changed from last time) An *isolated triangle* in G is a triangle with no other edge connecting one of its three vertices to a vertex not in the triangle. Let X be the number of triangles in $G_{n,p}$. For $p = c/n$, show that the number of isolated triangles becomes concentrated about its mean as $c \to \infty$, using Chebyschev's Inequality. That is, show for all $\epsilon > 0$, $\lim_{c\to\infty} \Pr(|X-\mu| \geq \epsilon\mu) = o(1)$.

2. Problems 2, 3, 5, 6, p. 58-9 of Alon and Spencer.

3. Problem 4, p. 36 of Alon and Spencer.

4. Problems 3, 8, p. 21 of Alon and Spencer.

5. Problems 4, 7, p. 11 of Alon and Spencer.

6. Show that for any n sufficiently large, there exists a graph G on n vertices with chromatic number at least $n/2$ with clique number at most $n^{3/4}$. Chromatic number is the smallest number k such that the vertices of the graph can be partitioned into k parts with no edges inside any part. Clique number is the size of the largest clique (complete subgraph) in the graph. (Hint: What can you say about the chromatic number of the complement of a triangle-free graph?)

Written problems for Wednesday, 4/12/06

Begin initial work on your project. Submit one or two pages summarizing the background material, including possibly important examples and counterexamples, and why the topic is important or useful.