MATH 100/ DS 100: Project on 'Orthogonality'1

Description: This project is about one of the most fundamental concepts in applied mathematics usually referred to as orthogonality.

You will investigate the following ideas, explain them, and illustrate them using Matlab:

- Cartesian coordinate systems in Euclidean spaces
 - Vectors
 - definition of angles via dot products and definition of orthogonality
 - orthogonal projections using angles and basic trigonometry
 - coordinates of vectors with respect to an orthogonal basis
 - generation of an orthogonal basis via the Gram-Schmidt method
- Orthogonality of functions
 - functions as "vectors"
 - "dot products" for functions and orthogonality of functions
 - orthogonal projections for functions
 - "coordinates" of functions with respect to an orthogonal basis
 - Gram-Schmidt for functions
- Applications of orthogonality such as Taylor polynomial approximation vs. orthogonal polynomial approximation, and Least squares approximation by orthogonal projections

The project requirements are:

- 1. Read and understand the above listed topics from textbooks or related online articles. Explain them to me.
- 2. Write a detailed report explaining the above with examples.
- 3. Illustrate these concepts by implementing examples in Matlab.
- 4. Give a presentation to your classmates on your project.

References: Some starting points for reading include a college Linear Algebra textbook such as for Math 332; or:

- (i) Orthogonality. In Wikipedia, The Free Encyclopedia.
- (ii) Gram-Schmidt process.. In Wikipedia, The Free Encyclopedia.
- (iii) Projection (linear algebra). In Wikipedia, The Free Encyclopedia.
- (iv) Austin, David (March 2009). No Static at All: Frequency modulation and music synthesis. Feature Column: Monthly essay on mathematical topics.

Retrieved from http://www.ams.org/publicoutreach/feature-column/fcarc-synthesizer.

(v) Victoria Hart [Vihart]. Aug 21, 2011. What's up with noises. Retrieved from YouTube http://youtu.be/i_0DXxNeaQ0.

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 $^{^1\}mathrm{Adapted}$ from Greg Fasshauer, CSM