MATH 380: Homework #5

Due Thursday, 2/19, 3pm, on Canvas via a single PDF.
NOTE THE TIME.

The homework exercises listed as numbers below are from the course textbook, Giordano, Fox, Horton, A First Course in Mathematical Modeling, 5th edition.

Follow the detailed instructions and rules for HWs given in the Course Information Handout and through Canvas and emailed comments.

Always remember that homework is NOT meant to be an examination, it is meant to assist in your learning and development. If you need help with any HW problem, don't hesitate to ask me. You are encouraged to ask questions during the *Class lectures*, through the *Canvas Discussion Forums*, during the *Office Hours*, during the *suggested TA office hours*, or through *Email to me*.

Submit solutions to each of the following problems. None of them require the use of computational tools.

1. Section 7.1: #4.

2. Submit one of the following two problems:

(a) We want to restrict a variable x to take values in a set $\{a_1, \ldots, a_m\}$ of numbers. Show how to do this using two linear constraints with binary variables.

(b) We are given two linear constraints $\mathbf{a}^T\mathbf{x} \geq b$ and $\mathbf{c}^T\mathbf{x} \geq d$ in which each component of both vectors $\mathbf{a}$ and $\mathbf{c}$ is nonnegative. (Recall that $\mathbf{a}^T\mathbf{x}$ means $\sum_{i=1}^{n} a_i x_i$, similarly $\mathbf{c}^T\mathbf{x}$.) The variable vector $\mathbf{x} \geq 0$ is required to satisfy **at least** one of the these two constraints. Show how to rewrite these linear constraints (without losing linearity) so that this requirement is satisfied. [*Comment*: You should not force $\mathbf{x}$ to satisfy both constraints always, there should be a choice of either satisfying only constraint number 1, or satisfying only constraint number 2, or satisfying both constraints.]

3. A hospital wants to make a weekly night shift (12am-8am) schedule for its nurses. The demand for the number of nurses on each night shift of the week is: 12 (Sun), 10 (Mon), 11 (Tues), 14 (Wed), 15 (Thurs), 18 (Fri), 17 (Sat). Each nurse works 5 days in a row on the night shift followed by two days off. Write an optimization model to find the minimum number of nurses the hospital needs to hire. (You do not have to solve it. If you are interested, try using a solver. Ask me for suggestions.)

4. Problem #3 from Sections 7.2 and 7.3.
[*Comment*: You will first write it as a Linear program and then solve it graphically by sketching the geometric feasible region and then (i) using family of lines corresponding to the objective function, and (ii) finding all the corner points.]