Teaching and Mentoring Statement

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Teaching Philosophy and Practice

I consider teaching a very important and enjoyable part of my academic life, and fortunately I have had the opportunity to teach a number of courses on a wide variety of topics, both elementary and advanced, at the University of Illinois and at Illinois Institute of Technology. There are many aspects to successful teaching, some of which might depend on the specifics of the course and the students taking it, but certain principles are essential to success in all forms of teaching. One of my primary goals as a teacher is to help students feel empowered to seek, learn, and apply knowledge. I encourage my students to defy low expectations, and to set and achieve substantial goals. My primary goal as a teacher is to push students to be intellectually ambitious and provide them the learning tools to achieve those high standards.

An essential ingredient for teaching well is the creation of a learning environment in class that makes students feel comfortable and that is conducive to doing mathematics. In such an atmosphere, it is easier to ask students to challenge themselves, and to process, critique and apply the concepts that they are taught. I want students to realize that math is not a spectator sport. I aim to teach them how to digest concepts and then apply them to problems. For example, when a student is stuck on a problem, I often ask if they can think of any examples or special cases to tackle first and help them reconnect the present problem to past concepts and problems. When I present an example, solution to a problem, and often, even a proof of a theorem, I first encourage them to think intuitively about possible approaches to an answer and then use that intuition to break the answer into solving a series of simpler problems that allows students to naturally discover the answer. With time, students also learn to use this method independently to understand concepts and ask better questions.

An obvious but very important aspect of learning in class are the instructor's lectures. For a good lecturing style, I try to present the course material clearly and at the appropriate speed so that students can understand the underlying theme as well as nitty-gritty of the details. I involve the class in the lecture by encouraging their questions and comments, and making my own. I spend a considerable amount of time planning and preparing the "core message" of a lecture. This could be the main result or methodology that is the least bit of information everyone in class should take home from the lecture. I also examine each concept, before teaching, for those aspects of it that students might find confusing. I address these difficulties as a part of the original exposition. When students do not completely grasp a concept or a problem at the first try, I encourage them to express the source of their confusion. Then, I revisit the problematic area to guide them through their unsuccessful attempts at learning and how to turn their "failure" into a learning experience.

Often, the class time is not enough to help all students. I utilize time outside class by encouraging students to send emails with their difficulties (and the majority of students regularly communicate in this manner with me); I put up additional lecture notes and examples on the course webpages; and I also hold weekly problem-solving sessions for some courses.

I emphasize logical thinking in class through examples and non-examples, and regular homework and exams that are meticulously evaluated. Building logical thinking is a semester-long (life-long) endeavor. Students perform better when they have an intuitive idea about a concept and can apply it rigorously. For example, in linear algebra, visualizing vectors pictorially can aid in understanding the various operations, like addition, associated with them and finding approaches to solving problems on them. Along the same lines, it is crucial that students have a sense of the "big picture" and how what they are being taught fits in this context. This "big picture" could involve different relevant fields, like economics or computer science, or simply be a related application area. If the students understand the relevance, importance and applicability of a topic, they are more motivated to learn and apply their knowledge in other areas as well.

Finally, I clearly state my grading and other policies in the beginning of the semester and adhere to them scrupulously. I also discuss my underlying philosophy for each course separately from the grading policies (you can find these documents on my webpage as well as some examples in appendices at the end of this document). I regularly assign homework with a variety of problems of different levels of difficulty, and discuss important or common conceptual difficulties therein during the class. I also encourage students to attend my office hours to discuss homework problems or any conceptual difficulties.

To extend learning beyond class time, I communicate with my students through a course webpage and regular email. In case of courses like Number Theory and Graph Theory, which are the first advanced proof-based courses for many students (math as well as engineering majors) at IIT, I hold weekly 'problem-solving' sessions outside class and upload additional examples related to that day's lecture on the course webpage. These have proven very popular with the students as a majority of them face difficulty making the transition to thinking and writing proofs (or simply making rigorous logical arguments). All these practices tie in to my previously stated philosophy of challenging students and encouraging them to be intellectually ambitious. Without these outside-the-class opportunities to extend their learning, it would not be feasible for me to challenge them or for them to rise up to the challenge. I have received a lot of positive feedback from students about this aspect of my courses - challenging but fair. And I am proud of that.

Course Development

One important aspect of teaching, that is finalized even before the teaching begins, is course development. The structure of a course – choice of topics and the amount of emphasis placed on each topic, can be a critical factor towards its success. I have been involved in curriculum development, even as a graduate student, through writing course proposals for graduate courses on topics that were not adequately covered at UIUC. My course proposal for a graduate course on discrete and convex geometry was accepted by the department and taught by Professor Füredi in Fall 2003. In 2005, I wrote a proposal for an advanced graduate course on probabilistic methods, like concentration of measure, entropy, and Markov Chain Monte Carlo, that are applied in graph theory, combinatorics, theoretical computer science, discrete optimization, and physics. This proposal was approved, and I was invited to teach this course in lieu of my TA duties. This interdisciplinary course attracted regular attendees among faculty (over 5) and graduate students (over 25) from mathematics and engineering departments. This gave me an opportunity to learn a lot about teaching an interdisciplinary course, for which no standard text exists, to students of varying backgrounds and abilities at a graduate level.

At IIT, I have had the opportunity to develop and teach a new course on Number Theory, to create and teach a new course on Linear Optimization on the request of Math and ECE departments, and to re-make from scratch the graduate course in combinatorics. I have used standard texts studied at top universities, as well as other lecture notes and books for additional material and non-standard problems, for the courses in Number Theory and Linear Optimization. The course on Number Theory goes from ancient Greek Mathematics to modern mathematics of the 19th century, from divisibility, congruences, distribution of prime numbers, number theoretic functions, diophantine equations and quadratic reciprocity to the 20th century applications of these concepts to cryptography. It is a mix of algorithms, proofs and applications that students from applied math as well as computer science find interesting.

The course on Linear Optimization introduces both theoretical and algorithmic aspects of linear optimization: geometry of linear programs, simplex method, anticycling, duality theory and dual simplex method, sensitivity analysis, large scale optimization via Dantzig-Wolfe decomposition and Bender's decomposition, interior point methods, network flow problems, integer programming, with many of these topics included at the request of faculty from ECE as relevant to their applications. I build a mathematical foundation through an interplay of geometry and linear algebra, that helps students understand the underlying concepts and apply them to modeling and algorithmic questions in non-trivial applications. Students have to do a semester-long project where they can apply these tools to their own research. This has proved very popular with students from engineering and computer science.

The graduate course in combinatorics, introduces students in applied mathematics, computer science, and engineering to tools and techniques from various fields of mathematics like probability, linear algebra, algebra, and stochastic processes, and applies them to existential and algorithmic problems arising in graph theory, combinatorics, and computer science. The tools include probabilistic methods, linear algebra methods, combinatorial nullstellensatz, entropy, martingales and large deviation bounds, Markov chain Monte Carlo, etc. These tools are applied to various fundamental problems like graph and hypergraph coloring, intersecting families of sets, Ramsey problems, extremal problems on graphs and on set systems (hypergraphs), optimization problems on discrete structures, sampling and counting discrete objects, etc. As there is no single textbook that covers all these topics, I have made my own lecture notes and problems for that course using results from research papers not found in any textbooks, and using multiple textbooks and lecture notes. There are very few universities around the world that might have such a one-semester introductory course on modern methods in discrete math.

Student Background and Feedback

My upper-level courses on Number Theory, Graph Theory, and Linear Optimization regularly attract many students from CS, ECE, and Civil Engineering departments in addition to the Applied Math students. Also, the Graph Theory and the Linear Optimization courses are dual-delivery courses with both undergraduate and graduate students. So these classes, even more so than usual, have a mix of both very talented students and some weak students, as the background of students and their mathematical maturity can vary drastically. The talented students need to be challenged and kept active with a variety of learning opportunities. While the weaker students have to be nurtured and brought up to pace with the rest of class. This bimodal distribution of abilities adds to the challenge of adapting my teaching style to suit students of varying academic backgrounds and different education levels. An easy way out is to lower the standards and cater to just getting students to somehow get through the course. I am strongly opposed to this. I maintain high standards in all my courses and challenge students to rise up to those standards. However, to help the weaker students keep up with the class I put in extra time with them outside class - in person and through email. I hold weekly problem-solving sessions outside class and upload additional examples/discussions related to that days lecture on the course webpage. For strong students, I give them opportunities to challenge themselves with carefully selected extra problems/ applications or reading material as per their choice.

I have followed similar principles of high standard, flexibility in teaching style, and accessibility to students in standard lower-level courses, with students from many departments, like those on Calculus and on Linear Algebra with great success (in terms of the improvement in the skills for mathematics and mathematical communication in even the weakest of students). The calculus courses are now using online automated HW software. While these online HW modules have made learning more accessible and flexible for students, there is the difficulty of ensuring that students are still learning to write meaningful/logical solutions to problems that others can also understand. To help students not lose track of how to think about and write solutions, I hold weekly written quizzes based on that week's HW. This helps students stay on top of the course and allows them to regularly practice communicating mathematics. I have taught the Linear Algebra course as a transition-to-proofs course. This makes it essentially the only such transition course in our department. The teaching of rigorous abstract concepts and their properties- vector spaces, inner product spaces, eigenvalues and eigenvectors, linear transformations, etc. - to students, who have only seen elementary methods-based courses so far, is a challenge. However, with a right mix of proofs, computational algorithms, and applications, students are motivated to make this transition into upper-level mathematics.

This flexibility in teaching style and adaptability to student abilities makes it more challenging to teach these courses but many students have told me that they find it refreshing to be challenged by my high standards while simultaneously being helped inside and outside the class to rise to those standards. My student course evaluations and the comments therein also attest to the same. I have received a per-course average student response of 4.52 out of 5 (since joining IIT) and 4.63 out of 5 (over the past 4 years) to "Overall, how would you rate this instructor?" in course evaluations with average response rate of 64% of registered students. My highest course evaluation was 4.92 for the Number Theory course in Fall 2010 (with a response rate of 12 out of 15 students).

In 2007, I was awarded the Project NExT fellowship by MAA which gave me an invaluable opportunity to discuss and learn even more about course development, teaching methodologies, and latest innovations from research in math education. Through all these experiences and practises over the past years, I hope that I am on the way to mastering the nuances of lecturing to students of differing experiences and capabilities, of managing courses at different levels, and of understanding the human as well as technical aspects of teaching and learning mathematics.

Student Mentorship

I have worked with 10 undergraduate students (ranging from Freshmen to Seniors) and 4 graduate students (2 completed M.S. Theses and 2 current Ph.D. students) on research in both Graph Theory and Optimization. Two of my students have won the top two awards from the Illinois Mathematical Association of America for best research by an undergraduate student. I have also worked with a talented high school student on original research in graph theory. IIT has a small Ph.D. program in Applied Math, where many of the students are interested in Financial or Computational Math, so I, as well as my other current colleagues in the discrete math group, have not had an opportunity to work with Ph.D. students yet. However, an incoming Ph.D. student in Fall 2011 will be working with me on his thesis research. In addition, we currently have two Ph.D. students in the first two years of their study who are interested in discrete math but haven't yet chosen their advisors. One of them, Jinyu Huang, is also starting his thesis research with me in Fall 2011.

Applied Math Freshmen in Fall 2006 and Fall 2008, did projects under my supervision that involved writing an expository report on self-discovered proofs and software development for 'Theory and Algorithms for Stable Matchings'. In Fall 2009 and Fall 2010, I gave a series of 3 lectures that introduces all the applied math freshman in Math 100 to different aspects of discrete mathematics through topics like graph coloring, visual combinatorial proofs, and planar graphs, and then tested their understanding through innovative problems. These projects and lectures have proven very popular and successful (as per the student feedback).

YoungJu Jo, an undergrad in Applied Math and ECE worked with me over a year (after taking my Graph Theory course) from summer 2008 to summer 2009 on a long-standing conjecture of Shermer from 1984 on guarding orthogonal art galleries with holes. She was able to use clever ideas based on graph coloring to substantially improve the best known result toward this conjecture and was awarded the First prize for 'Outstanding Undergraduate Research in Mathematics' by the Illinois Section of the Mathematical Association of America, in April 2009. She also presented her work at 5 different conferences and seminars in Spring 2009. She is finishing her M.S. at NYU.

Christos Mitillos, an undergrad in Applied Math and CS, worked with me on Fall-coloring of graphs, a common generalization of two hard problems of graph coloring and domination from summer 2008 to summer 2009. He was awarded the IIT College of Science and Letters' Summer Scholarship 2008 to work with me. And his original research garnered him the second prize for

'Outstanding Undergraduate Research in Mathematics' by the Illinois Section of the Mathematical Association of America, April 2009. He also gave presentations at 4 different conferences and seminars in Spring 2009. He has finished an M.S. in his home country of Cyprus and has joined IIT as a Ph.D. student in Fall 2011 to work with me. Cory Knapp, an undergrad in Applied Math, worked with me over spring and summer of 2010 on extending some of Christos' results on Fall-coloring of graphs with support from a Federal Work Study grant.

In fall 2010, I had the opportunity to work with a talented high school student, Derek Hardin from IMSA (Illinois Math and Science Academy). From Fall 2010 to Spring 2011, he spent almost every Wednesday in my office to study coloring problems related to graph products and ultimately was able to prove some new results related to the long-standing Hedetniemi-Lovasz Conjecture on coloring tensor product of graphs. He gave a poster and a presentation at the IMSA Colloquium. He is joining MIT for his undergraduate studies in Fall 2011. Cheng Chang, an undergrad in Applied Math, has been working with me over the summer and fall of 2011 to improve some of Derek's work. Unsuk Heo, an undergrad in ECE, is starting research with me in fall 2011.

Joseph Srigiri, a graduate student in applied math worked with me from fall 2008 to summer 2009 on his M.S. thesis. His work involved original research in Computational Discrete Geometry and Optimization under the title "Stochastic Models for the Art Gallery Problem: A Computational Study". His work was motivated by a variation of the art gallery problem that models the problem of placement of wireless routers (guards) in a hospital floor as a part of an interdisciplinary research proposal to study wireless communications in a hospital environment. He implemented algorithms for solving the art gallery problem by converting it into a discretized graph optimization problem and also developed algorithms for solving a 2-stage stochastic art gallery problem where the placement of guards (routers) is effected by the various possible changes to the floor plan due to temporary obstructions (called 'holes' in the art gallery literature). This new approach to art gallery problems combined techniques from graph optimization, integer programming, geometry, and stochastic programming. Joseph is currently working in industry.

Mary Fidler, a graduate student in applied math, worked with me from fall 2010 to summer 2011, on her M.S. thesis. Her work involved research on network flow algorithms and linear optimization. She developed and analyzed new algorithms for "The Simple Equal Flow Problem On Generalized Networks". Equal flow problem is the standard min-cost flow problem with additional non-network constraint that requires certain arc-flows to be equal. Combinatorial optimization algorithms have been developed for this problem. Mary was able to extend these algorithms for the same problem over generalized networks in which amount of flow leaving a node may not equal the amount of flow reaching the destination node. These optimization problems have applications in currency flows, water resource management, perishable commodity management, etc.

Two Ph.D. students are currently working under me, Jinyu Huang is starting his thesis work while Chris Mitillos is starting his Ph.D. program. Also two undergrads, one from ECE and one from Applied Math are also currently working on problems from grpah theory and optimization with ordinal references with me.

In addition to these research projects with students, I also conducted reading seminars with students on topics not covered in any courses in IIT. Hannah Kolb, an undergrad in applied math, studied advanced topics, like Graph Minors and Hadwiger's conjecture, and applications of the regularity lemma, in extremal graph theory with me over Fall 2009, and gave weekly lectures on what she learned and wrote very good lecture notes on the same. Hannah is a graduate student at UIUC now. In Fall 2009, another undergrad, Anusuya Dhewaju, did a reading course on Stochastic Optimization with me.

I also was a co-organizer of a reading seminar on discrete math with two other colleagues in IIT and DePaul from Fall 2006 to 2008. We studied topics, through papers, problems and presentations, that are not traditionally offered in courses at either of the universities with undergraduate and graduate students from IIT, UIC and DePaul. The topics included graph minor theory, combinatorial set theory, discrete geometry, topological methods in graph theory, etc. Many of the undergraduate students attending these weekly seminars ended up doing research in mathematics and going on to graduate school.

Appendix A: An Example of a Course Handout (part I)

COURSE INFORMATION: MATH 410 Number Theory
Fall 2010

Time and Place: 1:50pm, Tuesday and Thursday, at 124, Engineering 1 Bldg.

Instructor: Hemanshu Kaul

Office: 234B, Engineering 1.

Phone: (312) 567-3128

E-mail: kaul@math.iit.edu

Office Hours: Noon-1pm Tuesday and Thursday, and by appointment.
Emailed questions are also encouraged.

Course Communications: http://www.math.iit.edu/∼kaul/TeachingFall10/Math410.html
Check the course webpage regularly for homework assignments, announcements, and a lecture log (useful when you miss a class and when reviewing for an exam).
I often send emails with comments regarding HW problems, Exams, etc. Make sure your IIT email account is active and working.

Prerequisites: Math 230 Introduction to Discrete Mathematics. In particular, familiarity with proofs using induction, and elementary properties of integers.

Textbook: D. M. Burton, Elementary Number Theory, 6th ed., McGraw-Hill.
Each section covered in class must be fully read by you.

Course Description: This course has a two-fold aim (both equally important):
– Develop proficiency in concepts, theory, and applications of Number Theory, including divisibility, congruences, distribution of prime numbers, number theoretic functions, diophantine equations, applications to encryption methods; and
– Develop good habits of understanding, communicating, and writing proof-based mathematics.

Also see the separate document "My Aim for this Course".

An official description of the lecture topics and the course objectives is available at
"http://www.iit.edu/csl/am/programs/course_descriptions.shtml"

Grade Break-down: Homework worth 25%; Two mid-term exams worth 20% each ; Final exam worth 35% . The grading scale will be no more strict than A:85-100, B:75-84, C:65-74, D:55-64.

Class Attendance and Participation: The multitude of concepts introduced and developed in each class, as well as the importance of proofs in this course makes it critical to attend lectures and participate in class discussions. You are also expected to read the text, including reviewing the proofs done in class, and doing the examples not covered in class.

Examinations: The exam dates and their precise topics will be announced in class and on the course webpage. The final exam will be on all the topics covered during the semester. Make-up exams will be given only in case of a documented emergency.

Homework Assignment: Homework problems will be assigned once a week (typically on Thursday) which will be due one week later.
It is your responsibility to check the webpage for assignments and their due dates. Homework needs to be submitted at the beginning of class on the due date. It should be typed or written

legibly. Be sure to staple the pages together and write your name, course number, assignment number, and the date of submission on the front.

'Why and How' of Homework: Homework serves as an opportunity for students to practice communicating written mathematics with clarity of thought and language. In any course like this, learning good communication skills in mathematics is very important. As significant is the opportunity that a homework provides you to test your understanding of the material covered in class that week. Mathematics cannot be learned by listening or just reading a book - you have to **do** it. Considering the varying pace of learning of students in class and the lack of class time to explore every detail of every concept/Theorem, working through problems in the HW (both written and suggested problems) is an easy way for you to make sure that you are keeping up with the class. This is why homework is given a lot of importance in this course - dedicate enough time to it every week.

Some of the HW problems will be straightforward applications of the definitions or theorems studied in class, however every homework will also contain one or two slightly more challenging problems. Don't be disheartened if some problems take a while to solve. Such problems help develop your mathematical creativity. Discuss such problems with your classmates, and/ or ask me for help, but only after you have given them sufficient thought. Please remember that **homework is NOT meant to be an examination, it is meant to assist in your learning and development. If you need help with it, don't hesitate to ask.**

The following is important for the many non-computational exercises you will encounter in this course. To improve your mathematical writing quickly, start by writing draft solutions to homework early. A day or two later after you have had time to forget what you wrote, read it. If it doesn't make sense or convince you, rewrite it. Writing a solution requires saying what you mean and meaning what you say. Be intellectually honest. Intellectual dishonesty includes: 1) stating a "reason" without understanding its relevance. 2) Claiming a conclusion when you know you haven't proved it. 3) Giving an example and claiming you have proved the statement for all instances. **Include enough detail in your solutions so that your explanation is convincing to someone who hasn't thought about the problem before**. The proofs/ arguments should be presented so that your classmates could read them and follow the logic (step-by-step).

You are allowed to discuss homework problems with your classmates. However, the solutions should be written by you alone. Solutions for homework and exams must be written clearly, legibly, and concisely, and will be graded for both mathematical correctness and presentation. Points will be deducted for sloppiness, incoherent or insufficient explanation, or for lack of supporting rationale.

HELP: You are encouraged to ask questions during class, or in office hours, or through email. If you are having trouble solving a homework problem, I will be glad to direct you in the right direction. The same goes for any concept/ proof you have difficulty understanding. Don't hesitate to ask for help! I cannot help you if you don't take the initiative.

In the past, a lot of my students have regularly communicated with me over email. I encourage you to do the same, if that suits you better. You can also stop by my office after 4:30pm on Tuesdays and Thursdays, if I am available.

Reasonable accommodations will be made for students with documented disabilities. In order to receive accommodations, students must obtain a letter of accommodation from the Center for Disability Resources and make an appointment to speak with me as soon as possible. The Center for Disability Resources is located in the Life Sciences Building, Room 218, 312-567-5744 or disabilities@iit.edu.

Appendix B: An Example of a Course Handout (Part II)

What is this course *really* about?
aka My aim for this course (Math 410: Number Theory)

Hemanshu Kaul

According to Underwood Dudley, there are at least eight levels of mathematical understanding:

1. Being able to do arithmetic
2. Being able to substitute numbers in 'formulas'/ being able to state or use elementary properties of concepts
3. Given 'formulas'/ elementary properties of a concept, being able to get other 'formulas'/ elementary properties
4. Being able to understand hypotheses and conclusions of theorems
5. Being able to understand the proofs of theorems, step by step
6. Being able to *really* understand proofs of theorems: that is, seeing why the proof is as it is, and comprehending the underlying ideas of the proof and its relation to other proofs and theorems
7. Being able to generalize and extend theorems, and apply them to seemingly unrelated problems
8. Being able to see new relationships, and discover and prove entirely new theorems.

The word 'theorem' is used above in a very general sense - it can also represent algorithms and techniques with a mathematical basis.

Levels 5 and 6 would be considered basic mathematical ability for Math majors. Non-trivial applications of Mathematics would lie in-between levels 6 and 7. While levels 7 and 8 constitute research in Mathematics. A lot of engineering and physics is deep applied mathematics and requires understanding at or beyond levels 6 and 7.

Calculus courses focus on a mixture of 1 and 2. Math 230 (Introduction to Discrete Mathematics) focuses on 3 and 4. Math 332 (Matrices) focuses on 3 and 4 with a bit of 5 . In this course, the focus is more on the upper part of levels 3, 4, 5, and 6.

In the first half of the course, the focus is on 3, 4, and 5 under the context of very elementary properties of integers and solving linear congruences (essentially linear equations modulo an integer whose solutions are required to be integers), and systems of linear congruences. In the second half the focus shifts away from 3 and 4 towards 5 and 6. Along the way we will understand how and when quadratic congruences can be solved (unlike usual quadratic equations, they are hard to solve), properties of prime numbers (the building blocks of integers), algorithmic issues and applications related to these concepts, and famous unsolved problems.

For some of you this will be the first time proofs are featured so prominently in a course, and the first time you study abstract functions which can not be defined using a simple analytic formula.

Both these aspects are an important feature of all upper-level math courses. This course aims to help you transition to this new way of thinking and doing mathematics.

I hope this course will help you make progress through these levels of mathematical understanding, and mathematical maturity. I would consider this a successful course, if you gain confidence in your ability to read, understand, and write mathematical arguments (including proofs), especially as compared to the beginning of the semester. And, you have the confidence that you can read, understand, and apply any topic/ technique in Elementary Number Theory whenever you need it.

Appendix C: A Second Example of a Course Handout (Part I)

COURSE INFORMATION: MATH 554 Discrete Applied Mathematics II
(Mathematical Methods in Discrete Applied Mathematics)
Spring 2011

Time and Place: 11:25am, Monday and Wednesday, at 241, Engineering 1 Bldg.

Instructor: Hemanshu Kaul

Office: 234B, Engineering 1.

Phone: (312) 567-3128

E-mail: kaul@math.iit.edu

Office Hours: 1pm-2pm Monday and Wednesday, and by appointment.
Emailed questions are also encouraged.

Course Communications: http://www.math.iit.edu/∼kaul/TeachingSpr11/Math554.html
Check the course webpage regularly for homework assignments, announcements, and a lecture log (useful when you miss a class and when reviewing for an exam).
I often send emails with comments regarding HW problems, Exams, etc. Make sure your IIT email account is active and working.

Textbook: There is no required textbook. See below for a discussion of various possible textbooks.

Grade Break-down: Homework worth 35%; One mid-term exam worth 25%; Final exam worth 25%; and a project worth 15%. The grading scale will be no more strict than A:85-100, B:75-84, C:65-74, D:55-64.

Examinations: There will be a mid-term exam and a final exam. The exam dates and their precise topics will be announced in class and on the course webpage. Each of the exams might include a take-home component. Make-up exams will be given only in case of a documented emergency.

Project: Each student, in consultation with the instructor, will pick a topic or a research paper not covered in class. He/She will be expected to write a 5-6 page summary of the paper in their own words and present it in a 30 minute lecture in class. The summary should include a short background/ history of results in the area, description of the problem with examples, and overview of the results and the proof techniques used therein. The topic/ paper must be finalized by March 9th. All reports must be submitted by 27th April.

Prerequisites: You need to have some familiarity with the topics listed below. Don't worry if you don't know each and every topic listed below, in that case you just need to be willing to learn whenever something is needed in the course.

1. Graph Theory: Trees, bipartite graphs, spanning trees, independent set, clique, vertex and edge covers, connectivity, chromatic number, edge chromatic number. Books - *West, Intro to Graph Theory*; *Diestel, Graph Theory, 3rd ed.*

2. Combinatorics: Basic Counting (permutations and combinations, various ways of sampling, partitions of integers), pigeonhole principle, Inclusion-Exclusion principle. Books - any standard undergrad textbook like *Brualdi, Introductory Combinatorics*; *Roberts and Tesman, Applied Combinatorics*; *van Lint and Wilson, A Course in Combinatorics*

3. Linear Algebra: Vector spaces - definition and lots of examples, linear independence, spanning set, basis and dimension of a vector space, eigenvalues of a matrix. Book - any standard undergrad textbook.
4. Probability: Random variables (discrete and continuous), expectation, conditional expectation, variance, elementary properties of distributions like Bernoulli, Binomial, discrete uniform, continuous uniform, exponential, poisson, normal; discrete Markov chains. Books - any standard textbook like *Stirzaker, Elementary Probability*; *Grimmett and Stirzaker, Probability and Random Processes.*

Course Description - Topics and Textbooks: This graduate-level course in Discrete Mathematics will introduce students in Applied Mathematics, Computer science, and Engineering, to the use of tools and techniques from various fields of mathematics like Probability, Linear Algebra, Algebra, and Stochastic processes, to existential and algorithmic problems arising in Graph Theory, Combinatorics, and Computer science.

The tools considered would include Probabilistic Methods, Linear Algebra methods, Combinatorial Nullstellensatz, Entropy, Martingales and large deviation bounds, Markov chain Monte Carlo, etc. These tools will be applied to various fundamental problems like - Graph and Hypergraph coloring, Intersecting families of sets, Ramsey problems, Extremal problems on Graphs and on Set systems (Hypergraphs), Optimization problems on discrete structures, Sampling and counting discrete structures, etc.

There is no one textbook that covers all the topics that I plan to present. So, it will be important that you attend classes regularly and take lecture notes. *Jukna, Extremal Combinatorics with applications to computer science* contains material on Probabilistic, linear algebraic, and algebraic methods. *Habib, McDiarmid, Ramirez-Alfonsin, and Reed, Probabilistic methods for algorithmic discrete mathematics* has a lot of relevant survey articles.
For Probabilistic Methods, *Alon and Spencer, Probabilistic Method, 2nd (or 3rd) ed.* is the best reference.
For Linear Algebra methods, *Babai and Frankl, Linear Algebra Methods in Combinatorics* available at *http://www.cs.uchicago.edu/research/publications/combinatorics* is the standard reference.
For MCMC, *Levin, Peres, and Wilmer, Markov Chains and Mixing Times* is a good reference.

Homework: A total of 5-7 homework will be assigned over the semester. Each homework will be announced on the course webpage.

You are allowed to discuss homework problems with your classmates (and no one else). However, the solutions should be written by you alone. Any use of external help/ solutions, etc. will be considered a violation of IIT code of Academic Honesty and prosecuted accordingly. Solutions for homework and exams must be written clearly, legibly, and concisely, and will be graded for both mathematical correctness and presentation. Points will be deducted for sloppiness, incoherent or insufficient explanation, or for lack of supporting rationale.

To improve your mathematical writing quickly, start by writing draft solutions to homework early. A day or two later after you have had time to forget what you wrote, read it. If it doesn't make sense or convince you, rewrite it. Writing a solution requires saying what you mean and meaning what you say. Be intellectually honest. Intellectual dishonesty includes: 1) stating a "reason" without understanding its relevance. 2) Claiming a conclusion when you know you haven't proved it. 3) Giving an example and claiming you have proved the statement for all instances. **Include enough detail in your solutions so that your explanation is convincing to someone who hasn't thought about the problem before**. The proofs/ arguments should be presented so that your classmates could read them and follow the logic (step-by-step).

HELP: You are encouraged to ask questions during class, or in office hours, or through email.

If you are having trouble solving a homework problem, I will be glad to direct you in the right direction. The same goes for any concept/ proof you have difficulty understanding. Don't hesitate to ask for help! I cannot help you if you don't take the initiative.

In addition to office hours, you can also walk-in to my office (if I am available). In past, a lot of my students have regularly communicated with me over email. I encourage you to do the same, if that suits you better.

Reasonable accommodations will be made for students with documented disabilities. In order to receive accommodations, students must obtain a letter of accommodation from the Center for Disability Resources and make an appointment to speak with me as soon as possible.

Appendix D: A Second Example of Course Handout (Part II)

What is this course *really* about?
aka My aim for this course (Math 435/535: Linear Optimization)

Linear program is a kind of constrained optimization problem. A constrained optimization problem asks you to maximize or minimize a (objective) function of several variables when the variables have to satisfy certain (in)equalities (constraints). You have encountered constrained optimization problems in Multivariable Calculus when you studied the method of Lagrange Multipliers. Problems in linear optimization are both simpler and more complicated than such problems you have seen before. They are simpler because the objective function and the constraints all have to be linear. But they are complicated because they have lots of variables and many times have other constraints (like integrality) which make them computationally difficult. The assumption about linearity makes Linear Algebra (instead of calculus) the primary tool to tackle such problems. In fact, a general LP asks for a vector x that minimizes the dot product $\mathbf{c}.\mathbf{x}$ subject to $A\mathbf{x} \leq \mathbf{b}$.

Each inequality specifies points on one side of a hyperplane (a half-space) in $\mathbb{R}^n$, where n is the number of variables. The intersection of all these half-spaces gives a convex, n-dimensional polytope (the feasible set). Linearity of LP guarantees that the optimal solution occurs at a corner (vertex) of the polytope. So in principle, to solve an LP, all we have to do is find all vertices of this polytope and evaluate the objective function on each of these points and picking the smallest.

However, this is not a practical algorithm because the number of vertices can be large. For example, the following n constraints in n variables: $0 \leq x_i \leq 1$, give $[0,1]^n$, the n-dimensional hypercube as the polytope. And this hypercube has 2^n vertices. So it would take exponential number of steps to evaluate the objective function at all these possible solutions.

The simplex algorithm avoids this hassle by using the fact that if the objective function is not minimized at a particular vertex of the polytope then one of the neighboring vertices of the polytope will have a smaller objective function value. So, starting at a vertex we can take these "local" steps improving our objective function value and ultimately reach the optimal solution.

This is just the start of the story. Next obvious questions are: How do we pick a vertex to start our algorithm? How do we choose one of the neighboring vertices for the next step? Can we guarantee this will not take exponential number of steps? Can we easily check whether a particular solution is optimal or not? How can we solve very large LPs efficiently? How does the solution of the LP change if we change the objective function or the constraints by a small "amount"? How can we solve LPs that arise from Networks (like the Transportation problem)? How can we solve LPs that have the additional constraint that some of the variables must take integral value (Integer Programming)?

In this course, we will build a mathematical foundation through an interplay of geometry and linear algebra, that will allow us to understand all the underlying concepts of LPs stated above, and then explore the followup questions mentioned above. This will prepare you to apply these mathematical tools in non-trivial applications. In fact, Linear optimization techniques, including simplex method, are widely used in real-life applications for decision-making in logistics and supply chains for businesses in transportation and distribution industries (like airlines, retailers, grocery chains, etc.). Problems involving tens of thousand of variables and millions of constraints are solved regularly. As discussed below, such non-trivial applications of mathematics (like Linear Optimization techniques) requires a deep understanding of the underlying mathematical structures and concepts.

According to Underwood Dudley, there are at least eight levels of mathematical understanding:

1. Being able to do arithmetic
2. Being able to substitute numbers in 'formulas'/ being able to state or use elementary properties of concepts
3. Given 'formulas'/ elementary properties of a concept, being able to get other 'formulas'/ elementary properties
4. Being able to understand hypotheses and conclusions of theorems
5. Being able to understand the proofs of theorems, step by step
6. Being able to *really* understand proofs of theorems: that is, seeing why the proof is as it is, and comprehending the underlying ideas of the proof and its relation to other proofs and theorems
7. Being able to generalize and extend theorems
8. Being able to see new relationships, and discover and prove entirely new theorems.

The word 'theorem' is used above in a very general sense - it can also represent algorithms and techniques with a mathematical basis.

Levels 5 and 6 would be considered basic mathematical ability for Math majors. Non-trivial applications of Mathematics would lie in-between levels 6 and 7. While levels 7 and 8 constitute research in Mathematics. A lot of computer science, engineering, and physics is deep applied mathematics and requires understanding at or beyond levels 6 and 7.

Calculus courses focus on a mixture of 1 and 2. Math 230 (Introduction to Discrete Mathematics) focuses on 3 and 4. Math 332 (Elementary Linear Algebra) focuses on 3 and 4 with a bit of 5. In this course (Math 435/ 535), the focus is more on the upper part of levels 3, 4, 5, and 6, and the corresponding algorithms that arise out of this mathematical understanding. The aim is give you a firm foundation in levels up to 6, so that you can go onto levels 7 and 8, both as mathematicians and engineers (through non-trivial applications of Linear Optimization techniques).

I hope this course will help you make progress through these levels of mathematical understanding, and mathematical maturity. I would consider this a successful course, if you gain confidence in your ability to read, understand, and write mathematical arguments (including proofs), especially as compared to the beginning of the semester. And, you feel that you can read, understand, and apply any other topic/ technique in Linear Optimization that you might need later on in your career.

with best wishes,
Hemanshu Kaul